Error propagation in SMOS calibration subsystem

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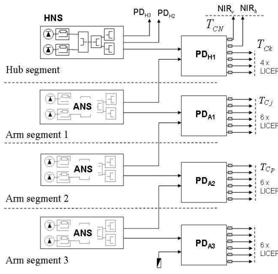
Ground characterization of Calibration Subsystem (CAS) of MIRAS (Microwave Imaging Radiometer using Aperture Synthesis)

- **CAS essential for overall performance of the payload**
- **Ground calibration of CAS enables calibration of receiver gain, T_{SYS} and phase imbalance**
- **The propagation and accumulation of errors in ground calibration to MIRAS calibration plane is simulated**
- **An analytical method of calculating error propagation is introduced**



CAS schematic

Amplitude calibration

gain of hub segment receivers from LICEF readings v_k and ground calibration S-parameters of paths *k, N*

$$G_{PMSk} = \frac{v_{k,hot} - v_{k,warm}}{\frac{|S_{kq}|^2}{|S_{Nq}|^2}\left(T_{CN}^{hot} - T_{CN}^{warm}\right)}$$

T_{CN} : noise levels measured by NIR at CAS outputs during noise injection.

→ gain of segment 1 receivers

$$G_{PMSj} = \frac{v_{j,hot} - v_{j,warm}}{\frac{|S_{jy}|^2}{|S_{ky}|^2}\left(\frac{v_{k,hot} - v_{k,warm}}{G_{PMSk}}\right)}$$

Phase calibration

phase imbalance of receivers *k* and *j* solved with ground calibration

$$\theta_{kj} = \theta'_{kj} - (\theta_{kq} - \theta_{jq})$$

similarly, for receivers *j* and *p*

$$\theta_{jp} = \theta'_{jp} - (\theta_{jq} - \theta_{pq})$$

thus for any receivers *k* and *p*

$$\theta_{kp} = \theta'_{kj} - (\theta_{kq} - \theta_{jq}) + \theta'_{jp} - (\theta_{jq} - \theta_{pq})$$

Calibrated gain, T_{sys} of receiver *k;* 6 NIR readings used for T_{diff}.

$$G_{PMS_k} = \frac{v_{k2} - v_{k1}}{\frac{|S_{k0}|^2}{6}\sum_{n=1}^{6}\frac{\left(T_{sys_n}^{C2} - T_{sys_n}^{C1}\right)}{|S_{n0}|^2}} = \frac{v_{k2} - v_{k1}}{\frac{1}{6}\sum_{n=1}^{6} X_{nk} T_{diff}^{n}},$$

$$X_{nk} = \frac{|S_{k0}|^2}{|S_{n0}|^2};\quad \Delta X_{nk} = 2\sqrt{\left(\frac{\Delta|S_{k0}|}{|S_{k0}|}\right)^2 + \left(\frac{\Delta|S_{n0}|}{|S_{n0}|}\right)^2} = 2\sqrt{2}\frac{\Delta|S|}{|S|}$$

$$T_{SYS,k} = \frac{v_A}{G_{PMS,k}}$$

Uncertainty of G_{PMS} due to ΔX is:

$$\Delta G_{PMS_k} = \sqrt{6 * \left(\frac{-T_{diff}^{n} T_{diff}^{k} G_{PMS,k}}{\frac{1}{6}\left(\sum_{n=1}^{6} X_{nk} T_{diff}^{n}\right)^2}\Delta X_{nk}\right)^2}$$

assuming $X_{nk} = 1$, and $T_{diff,n} = T_{diff,k}$

$$\Delta G_{PMS,k} = \frac{1}{\sqrt{6}} G_{PMS,k} \Delta X_{nk}$$

→Uncertainty of T_{SYS} calibration

$$\Delta T_{SYS,k} = \frac{v_A}{G_{PMS,k}^2} G_{PMS,k}\sqrt{\sum_{n=1}^{6}\left(\frac{1}{6^2}\right)\Delta X_{nk}^2}$$

finally

$$\Delta T_{SYS,k} = \frac{1}{\sqrt{6}} T_{SYS,k} \Delta X_{nk}$$

→ T_{SYS} expressed in terms of *one* amplitude measurement uncertainty

$$\frac{\Delta T_{SYS,k}}{T_{SYS,k}} = \frac{1}{\sqrt{6}} 2\sqrt{2}\left.\frac{\Delta|S|}{|S|}\right|_{nk} = \frac{2}{\sqrt{3}}\left.\frac{\Delta|S|}{|S|}\right|_{nk}$$

Phase imbalance of receivers *k* and *p,* in separate sections *m* and *l,* with section *j* in between

$$\theta_{kp} = \frac{1}{N}\sum_{i=1}^{N}\left(\theta_{ki}^{l} - \theta_{pi}^{m}\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(\theta_k - \left[\theta_{i_{CAS}}^{l} - \theta_{k_{CAS}}^{l}\right] + \theta_p - \left[\theta_{i_{CAS}}^{m} - \theta_{p_{CAS}}^{m}\right]\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(\theta_k + \theta_p\right)$$

$$+ \sum_{i=1}^{N}\left(\frac{1}{N}\theta_{i_{CAS}}^{m} - \frac{1}{N}\theta_{i_{CAS}}^{l} + \frac{1}{N}\theta_{k_{CAS}}^{l} - \frac{1}{N}\theta_{p_{CAS}}^{m}\right)$$

θp and θk unaffected by measurement accuracy of CAS.

For N receivers, remaining terms sum up to 2N+2 terms; assuming an identical error $\Delta\theta_{CAS}$ for values, RSS error in the calculated imbalance equals

$$\Delta\theta_{kp} = \sqrt{2N\left(\frac{1}{N}\Delta\theta_{CAS}\right)^2 + 2(\Delta\theta_{CAS})^2}$$

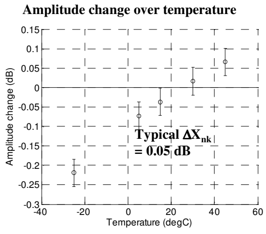
$$= \sqrt{\frac{2}{N} + 2}\cdot\Delta\theta_{CAS}$$

Calculation for multiple sections *C* between two receivers gives

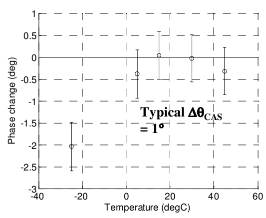
$$\Delta\theta_{kp} = \sqrt{\frac{2}{N}\cdot C + 2}\cdot\Delta\theta_{CAS}$$

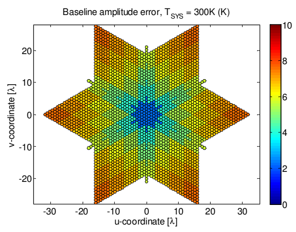
CAS characterization over temperature

Amplitude change over temperature



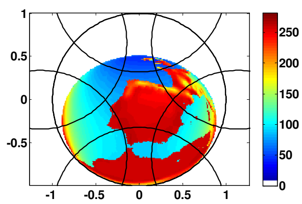
Phase change over temperature



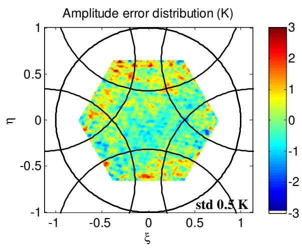


Calculated propagation of errors on MIRAS baseline chart

Standard deviation of error in alias-free zone
- **0.5 K for CAS amplitude errors**
- **0.9 K for CAS phase errors**
- **combined effect in MIRAS image 1.0 K**



Simulated MIRAS image using SEPS; including all error sources



Simulated MIRAS image using SEPS; effect of CAS amplitude errors